س1/ ضع كلا مما يأتي بالصيغة العادية للعدد المركب :

$$i^5 = i^4 \times i = 0 + i$$

$$i^6 = i^4 \times i^2 = -1 + 0i$$

$$i^{124} = (i^4)^{31} = 1 + 0i$$

$$i^{999} = (i^4)^{249} \times i^3 = 0 - i$$

$$i^{4n+1} = (i^4)^n \times i = 0 + i$$

$$(2 + 3i)^2 + (12 + 2i) = (4 + 12i - 9) + (12 + 2i)$$

$$= (-5 + 12i) + (12 + 2i) = 7 + 14i$$

$$(1 + i)^4 - (1 - i)^4 = ((1 + i)^2)^2 - ((1 - i)^2)^2$$

$$= (1 + 2i - 1)^2 - (1 - 2i - 1)^2 = (2i)^2 - (-2i)^2$$

$$= -4 + 4 = 0 + 0i$$

$$\frac{12 + i}{i} \times \frac{-i}{-i} = \frac{-12i - i^2}{1} = \frac{1 - 12i}{1} = 1 - 12i$$

$$\frac{3 + 4i}{3 - 4i} \times \frac{3 + 4i}{3 + 4i} = \frac{9 + 24i - 16}{9 + 16} = \frac{-7 + 24i}{25} = \frac{-7}{25} + \frac{24}{25}i$$

$$\frac{i}{2+3i} \times \frac{2-3i}{2-3i} = \frac{2i-3i^2}{4+9} = \frac{3+2i}{13} = \frac{3}{13} + \frac{2}{13}i$$

$$\left(\frac{3+i}{1+i}\right)^3 = \left(\frac{3+i}{1+i} \times \frac{1-i}{1-i}\right)^3 = \left(\frac{3-3i+i-i^2}{1+1}\right)^3 = \left(\frac{4-2i}{2}\right)^3$$

$$= \left(\frac{4}{2} - \frac{2i}{2}\right)^3 = (2-i)^3 = (2-i)^2(2-i)$$

$$= (4-4i-1)(2-i) = (3-4i)(2-i)$$

$$= (6-3i-8i-4) = 2-11i$$

$$\frac{2+3i}{1-i} \times \frac{1+4i}{4+i} = \frac{2+8i+3i-12}{4+i-4i+1} = \frac{-10+11i}{5-3i} \times \frac{5+3i}{5+3i}$$

$$= \frac{-50-30i+55i-33}{25+9} = \frac{-83+25i}{34} = \frac{-83}{34} + \frac{25}{34}i$$

$$(1+i)^3 + (1-i)^3 = (1+i)^2(1+i) + (1+i)^2(1+i)$$

$$= (1+2i-1)(1+i) + (1-2i-1)(1-i)$$

$$= 2i(1+i) + (-2i)(1-i) = 2i-2-2i-2$$

$$= -4+0i$$

س٢/ جد قيمة كل من x , y الحقيقيتين اللتين تحققان المعادلات الآتية :

$$y + 5i = (2x + i)(x + 2i)$$

$$y + 5i = 2x^2 + 4xi + xi - 2$$

$$2x^2 - 2 = y \qquad \ldots (1)$$

$$5x = 5 \Rightarrow x = 1 \qquad \ldots (2)$$

نعوض معادلة 2 في 1

$$2(1)^2 - 2 = y \Rightarrow y = 0$$

$$8i = (x + 2i)(y + 2i) + 1$$

$$8i = xy + 2xi + 2yi - 4 + 1$$

$$8i = xy + (2x + 2y)i - 3$$

$$xy - 3 = 0 \Rightarrow x = \frac{3}{y} \qquad \ldots (1)$$

$$2x + 2y = 8 \Rightarrow x = 4 - y \qquad \ldots (2)$$

نعوض معادلة 1 في 2 نحصل على :

$$\frac{3}{y} = 4 - y \Rightarrow 4y - y^2 = 3 \Rightarrow y^2 - 4y + 3 = 0$$

$$(y - 3)(y - 1) = 0$$

Either $\quad y - 3 = 0 \Rightarrow y = 3 \Rightarrow x = \frac{3}{3} = 1$

Or $\quad y - 1 = 0 \Rightarrow y = 1 \Rightarrow x = \frac{3}{1} = 3$

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$c)$ $\left(\dfrac{1-i}{1+i}\right) + (x+yi) = (1+2i)^2$

$\Rightarrow \left(\dfrac{1-i}{1+i} \times \dfrac{1-i}{1-i}\right) + (x+yi) = (1+4i-4)$

$\Rightarrow \left(\dfrac{1-i-i-1}{1+1}\right) + (x+yi) = (-3+4i)$

$\Rightarrow \left(\dfrac{-2i}{2}\right) + (x+yi) = (-3+4i) \Rightarrow -i + x + yi = -3+4i$

$\Rightarrow x = -3$

$\Rightarrow -1 + y = 4 \Rightarrow y = 5$

$d)$ $\dfrac{2-i}{1+i}x + \dfrac{3-i}{2+i}y = \dfrac{1}{i}$

$\Rightarrow \left(\dfrac{2-i}{1+i} \times \dfrac{1-i}{1-i}\right)x + \left(\dfrac{3-i}{2+i} \times \dfrac{2-i}{2-i}\right)y = \dfrac{1}{i} \times \dfrac{-i}{-i}$

$\Rightarrow \dfrac{2-2i-i-1}{1+1}x + \dfrac{6-3i-2i-1}{4+1}y = \dfrac{-i}{1}$

$\Rightarrow \dfrac{1-3i}{2}x + \dfrac{5-5i}{5}y = 0-i \Rightarrow \dfrac{1}{2}x - \dfrac{3}{2}xi + y - yi = 0 - i$

$\Rightarrow \dfrac{1}{2}x + y = 0$ نضرب طرفي المعادلة بـ2

$\Rightarrow x + 2y = 0 \Rightarrow x = -2y$ ...(1)

$$\Rightarrow -\frac{3}{2}x - y = -1 \qquad \text{نضرب طرفي المعادلة بـ2-}$$

$$\Rightarrow 3x + 2y = 2$$

$$\Rightarrow x = \frac{2 - 2y}{3} \qquad \text{...} (2)$$

نعوض معادلة (1) بمعادلة (2) نحصل على :

$$-2y = \frac{2 - 2y}{3} \Rightarrow -6y + 2y = 2 \Rightarrow -4y = 2 \Rightarrow y = -\frac{1}{2}$$

$$x = -2\left(-\frac{1}{2}\right) \Rightarrow x = 1$$

س3/ اثبت أن :

$$a) \quad \frac{1}{(2-i)^2} - \frac{1}{(2+i)^2} = \frac{8}{25}i$$

$$L.S \Rightarrow \frac{1}{(4 - 4i - 1)} - \frac{1}{(4 + 4i - 1)} = \frac{1}{3 - 4i} - \frac{1}{3 + 4i}$$

$$= \left(\frac{1}{3 - 4i} \times \frac{3 + 4i}{3 + 4i}\right) - \left(\frac{1}{3 + 4i} \times \frac{3 - 4i}{3 - 4i}\right) = \frac{3 + 4i}{9 + 16} - \frac{3 - 4i}{9 + 16}$$

$$= \frac{3 + 4i}{25} + \frac{(-3 + 4i)}{25} = \frac{8}{25}i = R.S$$

$b)$ $\dfrac{(1-i)^2}{1+i} + \dfrac{(1+i)^2}{1-i} = -2$

$L.S \Rightarrow \dfrac{(1-2i-1)}{1+i} + \dfrac{(1+2i-1)}{1-i}$

$= \left(\dfrac{-2i}{1+i} \times \dfrac{1-i}{1-i}\right) + \left(\dfrac{2i}{1-i} \times \dfrac{1+i}{1+i}\right) = \left(\dfrac{-2i-2}{2}\right) + \left(\dfrac{2i-2}{2}\right)$

$= -i - 1 + i - 1 = -2 = R.S$

$c)$ $(1-i)(1-i^2)(1-i^3) = 4$

$L.S \Rightarrow (1-i)(1-(-1))(1-(-i)) = (1-i)(2)(1+i)$

$= 2(1+1) = 2(2) = 4 = R.S$

س4/ حلل كلا من الاعداد 85 , 41 , 125 , 29 الى حاصل ضرب عاملين من الصورة $a + bi$ حيث

a, b عددان نسبيان.

$29 = (25+4) = (25-4i^2) = (5-2i)(5+2i)$

$125 = (100+25) = (100-25i^2) = (10-5i)(10+5i)$

$41 = (25+16) = (25-16i^2) = (5-4i)(5+4i)$

$85 = (81+4) = (81-4i^2) = (9-2i)(9+2i)$

س5/ جد قيمة x , y الحقيقيتين اذا علمت ان $\frac{6}{x+yi}$, $\frac{3+i}{2-i}$ مترافقان.

الحل :

$$\left(\overline{\frac{3+i}{2-i}}\right) = \left(\frac{6}{x+yi}\right) \Rightarrow \left(\frac{3-i}{2+i} \times \frac{2-i}{2-i}\right) = \left(\frac{6}{x+yi}\right)$$

$$\Rightarrow \frac{6-3i-2i-1}{4+1} = \frac{6}{x+yi} \Rightarrow \frac{5-5i}{5} = \frac{6}{x+yi}$$

$$\Rightarrow 1-i = \frac{6}{x+yi} \Rightarrow x+yi = \frac{6}{1-i} \times \frac{1+i}{1+i}$$

$$\Rightarrow x+yi = \frac{6+6i}{1+1} \Rightarrow x+yi = 3+3i$$

$$x = 3 \quad , \quad y = 3$$